VP_ Schaltung in SC-Technik.

开关电容电路. 之前讲解过原理, 现在结合信号与系统的 s 变换和 z 变换来研究

s 变换. $\sigma+j\omega$. 扩充函数到傅里叶变换不能满足的函数, 例如指数函数.

z 变换. $z=e^{sT} \Leftrightarrow s=\frac{1}{T}\ln z$, z 变换得到的是取样信号的双边拉普拉斯变换

1. Allgemeine Grundlagen.

主要用做采样和滤波

1.1. s 变换基础.

— 时域定理: $\mathcal{L}[f^{(1)}(t)] = sF(s) - f(0_-)$

— 积分定理: $\mathcal{L}[f^{(-1)}(t)] = \frac{1}{s}f^{(-1)}(0_-) + \frac{1}{s}F(s)$

— 初值定理: $f(0_+) = \lim_{s\to\infty} sF(s)$, $f'(0_+) = \lim_{s\to\infty} s[sF(s) - f(0_+)]$

— 终值定理: $f(t)\longleftrightarrow F(s)$, $f(\infty)=\lim_{s\to 0} sF(s)$

常用的 s 变换:

$$e^{at}\varepsilon(t) \longleftrightarrow \begin{cases} \frac{1}{s-a}, & \mathrm{Re}[s]=\sigma>a. \\ \text{无界}, & \sigma<a. \\ \text{不定}, & \sigma=a. \end{cases} \qquad e^{\beta t}\varepsilon(-t) \longleftrightarrow \begin{cases} \text{无界}, & \mathrm{Re}[s]=\sigma>\beta \\ \text{不定}, & \sigma=\beta \\ -\frac{1}{s-\beta}, & \sigma<\beta \end{cases}$$

$g_\tau(t-\frac{\tau}{2}) \longleftrightarrow \frac{1-e^{-s\tau}}{s}$, $\mathrm{Re}[s]>-\infty$, $\delta(t)\longleftrightarrow 1$, $\delta'(t)\longleftrightarrow s$, $\varepsilon(t)\longleftrightarrow\frac{1}{s}$ $\mathrm{Re}[s]>0$, $e^{j\beta t}\longleftrightarrow\frac{1}{s-j\beta}$, $\mathrm{Re}[s]>0$

其余性质见 PDF → P238.

电路的 s 域模型, 由 $\Sigma I=0$, $\Sigma U=0$ 建立方程求解.

(1) 电阻 R $(\frac{1}{G})$ $\quad U(s)=RI(s)$, $I(s)=GU(s)$

(2) 电感 L. $\quad u=L\frac{di}{dt}$ $\quad U(s)=sLI(s)-Li_L(0_-)$, $I(s)=\frac{1}{sL}U(s)+\frac{i_L(0_-)}{s}$

(3) 电容 C. $\quad i=C\frac{du}{dt}$ $\quad U(s)=\frac{1}{sC}I(s)+\frac{u_C(0_-)}{s}$ $\quad I(s)=sCU(s)-Cu_C(0_-)$

对于电路使用串联等效或并联等效计算.

1.2. z 变换.

常用 z 变换.

$$a^k\varepsilon(k) \longleftrightarrow \begin{cases} \frac{z}{z-a}, & |az^{-1}|<1 \\ \text{不定}, & |\frac{a}{z}|=1 \\ \text{无界}, & |\frac{a}{z}|>1. \end{cases} \qquad b^k\varepsilon(-k) \longleftrightarrow \begin{cases} \frac{-z}{z-b}, & |z|<|b| \\ \text{不定}, & |z|=|b| \\ \text{无界}, & |z|>|b| \end{cases}$$

$\varepsilon(k)\longleftrightarrow\frac{z}{z-1}$, $|z|>1$ $\qquad \varepsilon(-k-1)\longleftrightarrow\frac{-z}{z-1}$, $|z|<1$.

性质. PDF – P292.

— 双边变换 $f(k\pm m)\longleftrightarrow z^{\pm m}F(z)$

— 单边 $f(k-m)\longleftrightarrow z^{-m}F(z)+\sum_{k=0}^{m-1}f(k-m)z^{-k}$

— z 域微分. $k^m f(k)\longleftrightarrow\left[-z\frac{d}{dz}\right]^m F(z)$

— z 域积分 $\frac{f(k)}{k+m}$, $k+m>0 \longleftrightarrow z^m\int_z^\infty\frac{F(\eta)}{\eta^{m+1}}d\eta$.

— 初值定理. $f(0)=\lim_{z\to\infty}F(z)$

$f(m)=\lim_{z\to\infty}[F(z)-\sum_{k=0}^{m-1}f(k)z^{-k}]$

— 终值定理 $f(\infty)=\lim_{z\to 1}\frac{z-1}{z}F(z)$

1. 欧拉方法. (近似理解为迭代, 较深奥)

- 前进. $z=e^{sT_s}\approx 1+sT_s \rightarrow sT_s=z-1$

- 后退. $z^{-1}=e^{-sT_s}=1-sT_s \rightarrow sT_s=1-z^{-1}$

2. 双线性近似.

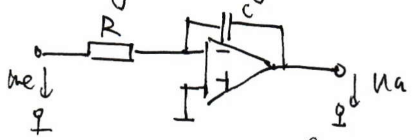
$sT_s=\ln z=\frac{z-1}{z+1}\cdot 2.$

$z=e^{sT_s}=\frac{e^{\frac{sT_s}{2}}}{e^{-\frac{sT_s}{2}}}=\frac{1+\frac{sT_s}{2}}{1-\frac{sT_s}{2}}$

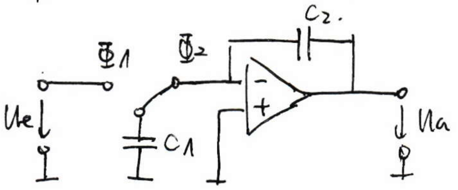
$\Rightarrow$

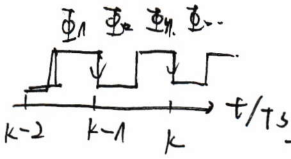
$sT_s=\frac{z-1}{z+1}\cdot 2$

analoger Integrator. ，反馈接电阻会影响开环增益，接电容对低频小信号几乎无影响；



$$G(s) = \frac{U_a(s)}{U_e(s)} = \frac{-\frac{1}{sC} I(s)}{R I(s)} = -\frac{1}{s\tau}$$ ，这里是积分器





这里的电路是一个数控态的电路，前提是保持了瞬态过程在采样时已经完成了。实际上时钟是有一定的死区时间，所以是可以工作的。

Φ1时，C_1充电，Φ2时，C_1上电压为0，电荷全部到C_2，可以通过电荷守恒计算。

allgemein

$$\sum_n C_n [U_{cn}(\Phi_1) - U_{cn}(\Phi_2)] = 0$$

这里 $C_1 \cdot U_{c1}(\Phi_1) + C_2 U_{c2}(\Phi_1) = C_1 U_{c1}(\Phi_2) + C_2 U_{c2}(\Phi_2)$

$C_1 U_e(k-1) + C_2 [-U_a(k-1)] = 0 - C_2 U_a(k)$

采用双边Z变换.

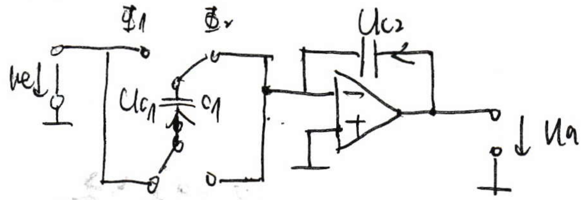
$$C_1 \frac{1}{z} U_e(z) - C_2 \frac{1}{z} U_a(z) = -C_2 U_a(z) \Rightarrow U_a(z) = -\frac{C_1}{C_2} \cdot \frac{1}{z-1} U_e(z)$$

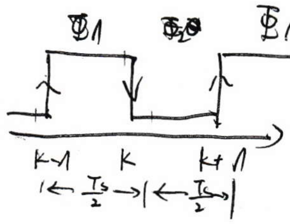
优点：电容好做、时钟可调。缺点：时钟频率要高，信号有失真和衰减

9.2. SC-Integratoren.

9.2.1. 欧拉方法

9.2.2. 双线性近似





列出方程. $C_1[-U_e(k-1) - U_e(k)] + C_2[U_a(k-1) - U_a(k)] = 0$

$\cancel{-C_1 U_e(z) - C_1 U_e(z)}$

$$-C_1[z^{-1} U_e(z) + U_e(z)] + C_2[z^{-1} U_a(z) - U_a(z)] = 0$$

$$U_a(z) C_2 \left(\frac{1}{z} - 1\right) = C_1\left(\frac{1}{z} + 1\right) U_e(z) \Rightarrow U_a(z)/U_e(z) = -\frac{C_1}{C_2} \cdot \frac{z+1}{z-1}.$$

代入 $sT_s = 2\frac{z-1}{z+1} \Rightarrow G(s) = -\frac{C_2}{C_1} \cdot \frac{2}{sT_s} = -\frac{1}{s\tau}$，其中 $\tau = \frac{C_1}{C_2} \cdot \frac{T_s}{2}$.

9.3. SC-Biquad-Filter.

二阶电路、通常系统的G变换为：

$$G(s) = -\frac{k_0 + k_1 s + k_2 s^2}{\omega_p^2 + \frac{\omega_p}{Q} s + s^2}, \quad G(s_0) = -\frac{a_0 + a_1 s_0 + a_2 s_0^2}{1 + \frac{s_0}{Q} + s_0^2} = \frac{U_a(s_0)}{U_e(s_0)}, \quad s_0 = \frac{s}{\omega_p}$$

通过数值来判定滤波器的类型。$a_1 = 0$，$a_2 = 0$，则为低通

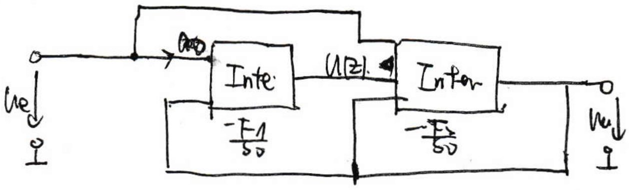
双二阶滤波器，对于不同的Q值未必会影响二阶低通滤波器的衰减

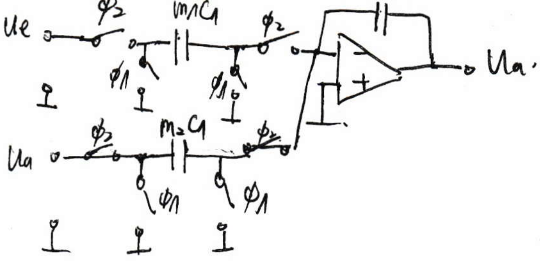
$$U_a(s_0)\left[1 + \frac{s_0}{Q} + s_0^2\right] = -[a_0 + a_1 s_0 + a_2 s_0^2] U_e(s_0)$$

$U_a(s_0) = -\left[U_a(s_0) \cdot \frac{s_0}{Q} + U_a(s_0) \cdot s_0^2 + a_0 U_e(s_0) + a_1 s_0 U_e(s_0) + a_2 s_0^2 U_e(s_0)\right]$，同除 s_0^2，之后 $U_a(s_0)$ 和 $s_0^2 U_a(s_0)$ 互换

$$= -\frac{1}{s_0}\left[\frac{U_a(s_0)}{Q} + (a_1 + a_2 s_0) U_e(s_0) + \underbrace{\frac{1}{s_0}(a_0 U_e(s_0) + U_a(s_0))}_{-U_2(s_0)}\right]$$

$-\frac{1}{s_0}$ 代表一个积分电路





SC-01.

每段时间 U_a 可以认为不变.
电荷量变化为电流的积分. 所以方程成立.

(1). ϕ_1: 只有 C_3, C_2, 抹掉 ϕ_2: ~~电流方程~~ 加入 C_1.

$$\Rightarrow C_3U_{C3}(\phi_1)+C_2U_{C2}(\phi_1)=C_3U_{C3}(\phi_2)+C_2U_{C2}(\phi_2)+C_1U_{C1}(\phi_2)$$

$$\Rightarrow -C_3U_{e2}(\phi_1)-C_2U_{a}(\phi_1)=-C_3U_{e2}(\phi_2)-C_2U_a(\phi_2)-C_1U_{e1}(\phi_2)$$

$\phi_1 \to k-1 \qquad \phi_2 \to k$.

$$\Rightarrow \frac{1}{z}C_3U_{e2}(z)+\frac{1}{z}C_2U_a(z)=C_3U_{e2}(z)+C_2U_a(z)+C_1U_{e1}(z)$$

$$-\left(1-\frac{1}{z}\right)C_2U_a(z)=\left(1-\frac{1}{z}\right)C_3U_{e2}(z)+C_1U_{e1}(z)$$

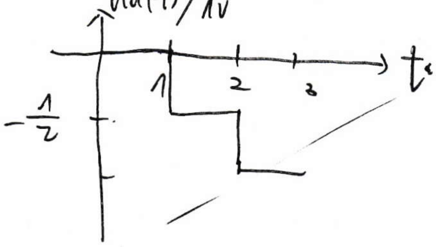
$$U_a(z)=-\frac{C_3}{C_2}U_{e2}(z)-\frac{C_1}{C_2}\frac{1}{\frac{z-1}{z}}U_{e1}(z)$$, 可以直接写出结果.

Nach EULER Rückwärts $sT_S=1-z^{-1}=\frac{z-1}{z}$

$$U_a(s)=-\frac{C_3}{C_2}U_{e2}(s)-\frac{C_1}{C_2}\frac{1}{sT_S}U_{e1}(s)$$

(2) ~~...~~ Gilt $U_a(s)=-\frac{C_1}{C_2}\cdot\frac{1}{sT_S}U_{e1}(s)=-\frac{1}{s\tau}U_{e1}(s)$, $\tau=\frac{C_2\cdot T_S}{C_1}$

$\Rightarrow$ mit $\tau=T_S\frac{C_2}{C_1}=100\,\mu s$ $\qquad U_a(t)=-\frac{t}{\tau}U_{e1}(t)$



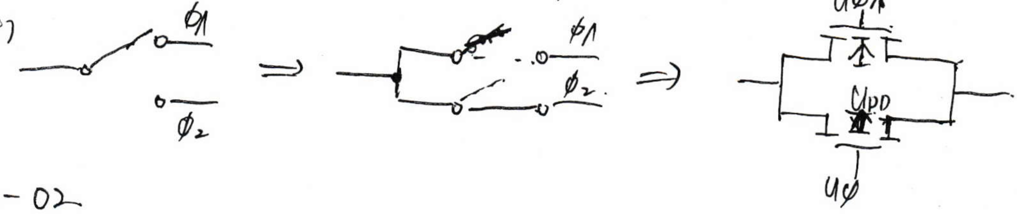
(3) 先只考虑 U_{e1}.

$$C_2U_a(z)=C_2\cdot\frac{1}{z}\cdot U_a(z)+C_1\cdot\frac{1}{z}U_{e1}(z)$$

$$\Rightarrow \frac{z}{z-1}\cdot\frac{C_1}{C_2}\cdot U_{e1}(z)=U_a(z)$$

$$\Rightarrow U_a(z)=-\frac{C_3}{C_2}U_{e2}(z)+\frac{C_1}{C_2}\frac{1}{z-1}U_{e1}(z)$$, mit Vorwärts. $sT_S=z-1$

(4)



SC-02

(1). $C_1U_{c1}(\phi_1)+C_2U_{c2}(\phi_1)=C_1U_{c1}(\phi_2)+C_2U_{e2}(\phi_2)$

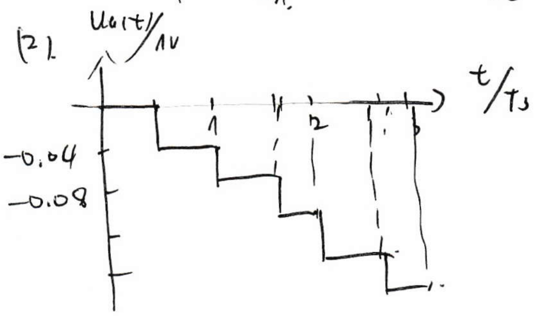
$$-C_1U_e(k-1)+C_2U_a(k-1)=C_1U_e(k)+C_2U_a(k)$$

$$U_a(z)\,C_2\left(1-\frac{1}{z}\right)=C_1\left(1+\frac{1}{z}\right)U_e(z)$$

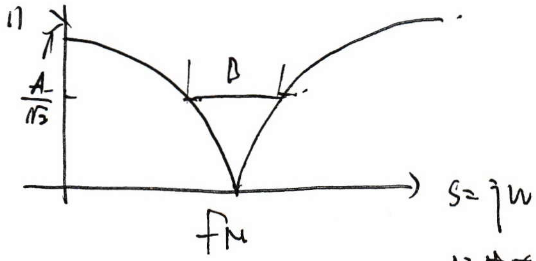
$$\frac{U_a(z)}{U_e(z)}=\frac{-C_1(z+1)}{C_2(z-1)}=-\frac{C_1}{C_2}\cdot\frac{z+1}{z-1}=-\frac{C_1}{C_2}\cdot\frac{4}{sT_2}\cdot\frac{sT_0}{2}=\frac{z-1}{z+1}\cdot\ldots,\quad T_S=\frac{T_0}{2}$$

$$\Rightarrow \tau=\frac{T_S}{4}\cdot\frac{C_2}{C_1}\quad\frac{1}{\tau}=0.04\frac{1}{\mu s}$$

(2).



SC-03.



$s_0 = \frac{s}{\omega_p}$, für $s = s_N$: $s_0 = j(\omega_N / \omega_p)$ ~~$s_N = j\omega_p$~~

$\Rightarrow s_0 = j$, ∵ $G(s_0) = 0$.

$\Rightarrow G(s_0 = j) = \frac{a_0 + a_1 j + a_2 j^2}{1 + \frac{j}{Q} + j^2} = 0 \Rightarrow jQ(a_0 - a_2 + j a_1)$

$\Rightarrow a_1 = 0,\ a_0 = a_2$.

题中有 $G(0) = -1$, $a_0 = a_2 = 1$, $a_1 = 0$.

$f_{s_0} = \frac{f_s}{\omega_p}$ $\quad T_{s_0} = \frac{\omega_p}{f_s} = \frac{f_N \cdot 2\pi}{f_s} = 0.1171$

(2) $\underline{U}_a(s_0) = \frac{1}{s_0}\left[s_0 \underline{U}_e(s_0) + \frac{1}{s_0}\left[\underline{U}_e(s_0) + \left(1 + \frac{s_0}{Q}\right) \underline{U}_a(s_0) \right]\right]$

$\underline{U}_Z = -\frac{1}{s_0}\left[\underline{U}_e(s_0) + \left(1 + \frac{s_0}{Q}\right) \underline{U}_a(s_0) \right]$. , 套用 SC-01. (3)

$\underline{U}_a(s_0) = -\frac{1}{s_0}\left[s_0 \underline{U}_e(s_0) - \underline{U}_Z(s_0) \right]$. 套用 SC-01 (1)

$s_0 = s\tau \rightarrow \frac{1}{s_0} = \frac{1}{s}\cdot\frac{1}{\tau} = \frac{1}{s}\omega_p = \frac{1}{s}\cdot\frac{T_{s_0}}{T_s}$

1.(1) $\underline{U}_a(s) = -\frac{C_3}{C_2}\underline{U}_{e2}(s) - \frac{C_1}{C_2}\frac{1}{sT_s}\underline{U}_{e1}(s)$ $\Rightarrow$

1.(3) $\underline{U}_a(s) = -\frac{C_3}{C_2}\underline{U}_{e2}(s) + \frac{C_1}{C_2}\frac{1}{sT_s}\underline{U}_{e1}(s)$ $\Rightarrow$

$P_1 C_1$ =

$P_2 C_1$

$P_3 C_1 \Rightarrow P_2 = P_3$

$\frac{C_1}{C_2} = 1 + \frac{s_0}{Q}$

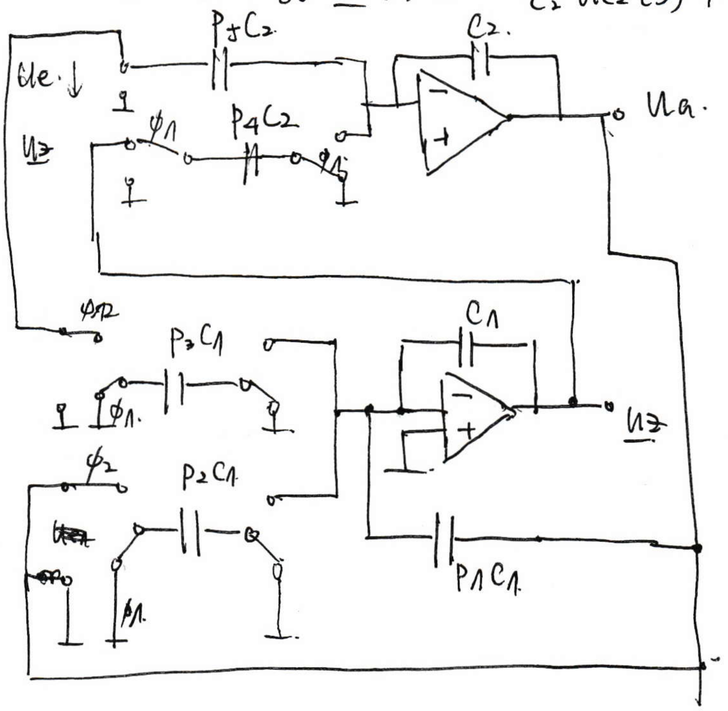
$\frac{C_3}{C_2} = 1$, $U_{e2} = U_e$, $U_{e1} = U_a$.

$P_4 C_2$

$P_5 C_2$.

$-\frac{1}{s_0}\left[\underline{U}_e(s_0) + \underline{U}_a(s_0) \right] - \frac{1}{Q}\underline{U}_a(s_0) = -\frac{C_3}{C_2}\underline{U}_{e2}(s) - \frac{C_1}{C_2}\cdot\frac{1}{sT_s}\underline{U}_{e1}(s)$

$-\underline{U}_e(s_0) + \frac{1}{s_0}\underline{U}_Z(s_0) = -\frac{C_3}{C_2}\underline{U}_{e2}(s) + \frac{C_1}{C_2}\frac{1}{sT_s}\underline{U}_{e1}(s)$



$P_1 = \frac{1}{Q}$

$P_2 = P_3 = T_{s_0}$.

$P_4 = T_{s_0}$

$P_5 = 1$.