6. Rauschanalyse.

1.1. 由于热噪声可以由电压源外加电流源来等效代替，对于一个线性系统来说

$\underline{S_y} = \underline{G}\,\underline{S_x}\,\underline{G}^+$. $S_y(f) = S_x(f)|H(f)|^2$.

S_x 是输入的噪声矩阵

$\underline{G}$ 传输矩阵.

$\underline{G}^+$ 传输矩阵的共轭转置矩阵(或共轭)

"我"认为 $P_N = G\int_{f_1}^{f_2} S_u(f)\,df$. Leitwert. G为电导率

$SNR = \frac{P_S}{P_N} = \frac{U_s^2}{2}/BS_u$, $SNR_{dB} = 10\lg(SNR)$ dB.

6.2. 电路中的电阻噪声模型

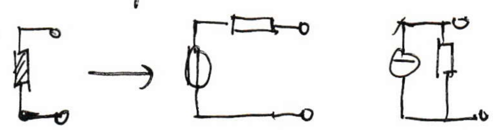
6.2.1. 散粒噪声：由电流的涨落引起.

$S_{IN} = 2eI$. e为电子的电荷

6.2.2. 热噪声.

粒子则为视电子，符合玻尔兹曼—爱因斯坦分布

$S_P = kT$.



$S_P = kT = \frac{U^2}{2}/R = \frac{I^2}{2}R$ 由此功率源最大可以输出的噪声功率源为 kT.

$S_{UN} = 4kTR$.

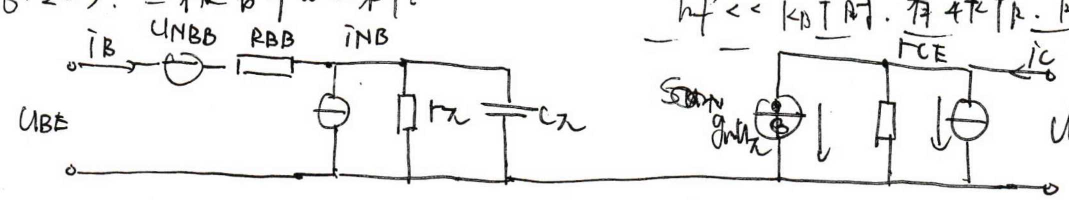
$S_{IN} = 4kTG$. 与电阻值无关

可以参考 MT2-VL6 的杂查斯特的模型。可以用2分式，将电阻分成阻抗正成的两个电阻 R1, R2，从而得出结论

白噪声通过线性网络之后一般都不是白噪声了

$hf \ll kT$ 时，有 4kTR. 所以是最大噪声功率.

6.2.3. 三极管中的噪声

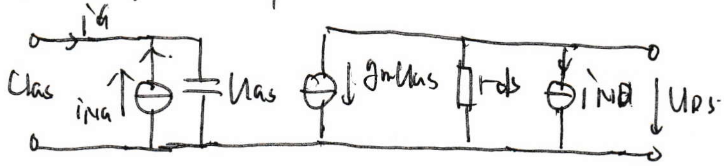


$r_\pi = \frac{U_T}{I_{BA}}$ $g_m = \frac{I_{BA}}{U_T}$

三极管中的是散粒噪声

$S_{UNBB} = 4kTR_{BB}$ $S_{INB} = \frac{2kT}{r_\pi}$ $S_{INC} = 2kTg_m$

6.2.4. 场效应管的噪声.



$S_{IND} = 4kT\gamma g_{d0}$， γ 和沟道效应系数 γ 特点相同，但含义不同。 < 饱和区(可变)：1. 恒流区：2/3.

$g_{d0} = g_{DS}|_{U_{DS}=0}$.

$S_{ING} = 4kT\delta g_G$， $\delta = 2\gamma$， $g_G = \frac{(\omega C_{GS})^2}{5 g_{d0}}$， 此时可以将 S_{ING} 认为是闪烁噪声.

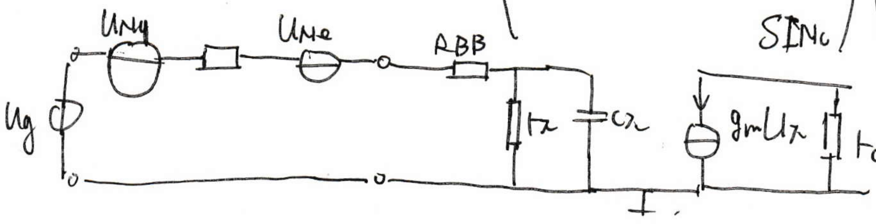
相关系数.

$\underline{c_g} = \frac{S_{GD}}{\sqrt{S_{ING}\cdot S_{IND}}} = 0.3j$ 了.

6.3. Rausch in Schaltungen.

$$S_{UNe} = (\underline{G_1}, \underline{G_2}, \underline{G_3}) \begin{pmatrix} S_{UNBB} & & \\ & S_{INB} & \\ & & S_{INC} \end{pmatrix} \begin{pmatrix} \underline{G_1}^* \\ \underline{G_2}^* \\ \underline{G_3}^* \end{pmatrix}$$

说明这里是三极管



求一下3个系数.

① $G_1 = 1$.

② 用 U_π 建立方程，求解系数.

$$U(\pi)\Big|_{U_{NB}} = \frac{r_\pi // \frac{1}{sC_\pi}}{(R_g+R_{BB}) + r_\pi // \frac{1}{sC_\pi}} \cdot U_{NB}$$

$$U(\pi)\Big|_{I_{NB}} = -I_{NB}\cdot\left[(R_g+R_{BB}) // r_\pi // \frac{1}{sC_\pi}\right]$$

$$= I_{NB}\cdot\frac{(R_g+R_{BB})\cdot r_\pi // \frac{1}{sC_\pi}}{(R_g+R_{BB}) + r_\pi // \frac{1}{sC_\pi}}$$

$$G_2 = \frac{r_\pi // \frac{1}{sC_\pi}\cdot (R_{BB}+R_g)}{(R_g+R_{BB}) + r_\pi // \frac{1}{sC_\pi}} \cdot \frac{R_g+R_{BB}+r_\pi // \frac{1}{sC_\pi}}{r_\pi // \frac{1}{sC_\pi}} = -(R_{BB}+R_g)$$

③ $U(\pi)\Big|_{I_{NC}} = \frac{1}{g_m}\left(i_{NC} - \frac{b\cdot U_\pi}{r_{CE}}\right) \Rightarrow g_m U_\pi + \frac{b}{r_{CE}}U_\pi = I_{NC} \Rightarrow U_\pi = I_{NC}\cdot\frac{1}{g_m+\frac{b}{r_{CE}}}$

$$\Rightarrow \frac{1}{g_m+\frac{b}{r_{CE}}}\cdot\left(\frac{R_g+R_{BB}}{r_\pi // \frac{1}{sC_\pi}} + 1\right)$$

$$r_\pi = \frac{U_T}{I_{BA}} = \frac{U_T}{\frac{I_{CA}}{b}} = b\frac{1}{g_m}$$

$$= \frac{1}{g_m+\frac{b}{r_{CE}}} + \frac{R_g+R_{BB}}{g_m+\frac{b}{r_{CE}}} \Big/ \frac{r_\pi\cdot\frac{1}{j\omega C_\pi}}{r_\pi+\frac{1}{j\omega C_\pi}}$$

$$= \frac{1}{g_m+\frac{b}{r_{CE}}} + \frac{R_g+R_{BB}}{g_m+\frac{b}{r_{CE}}}\cdot\frac{1+r_\pi\cdot j\omega C_\pi}{r_\pi}$$ 忽略 r_{CE}.

$$= \frac{1}{g_m} + \frac{(R_g+R_{BB})(1+r_\pi j\omega C_\pi)}{\frac{I_{CA}}{U_T}\cdot\frac{U_T}{I_{BA}}} = \frac{1}{g_m} + \frac{(R_g+R_{BB})(1+r_\pi j\omega C_\pi)}{b}$$

代入系数. $\omega = 0$.

$$S_{UNe}(\omega) = S_{UNBB}\cdot|G_1|^2 + S_{UNB}|G_2|^2 + S_{UNIC}|G_3|^2$$

$$= 4kT\left[R_{BB} + \frac{g_m}{2b}\cdot(R_g+R_{BB})^2 + \frac{g_m}{2}\cdot\left(\frac{1}{g_m}+\frac{R_g+R_{BB}}{b}\right)^2\right]$$

6.3.2. 噪声系数、

定义：~~输入端的信号功率~~ $NF = \frac{(S/N)_i}{(S/N)_o} = \frac{S_i/N_i}{S_o/N_o}$. 用线性网络传输系数定为

$K_p = \frac{S_o}{S_i}$, $\Rightarrow NF = \frac{N_o S_i}{N_i S_o} = \frac{N_o}{N_i\cdot K_p} = \frac{K_p N_i + N_a}{N_i\cdot K_p} = 1 + \frac{N_a}{N_i\cdot K_p} = 1+\frac{N_a}{N_o} = 1+\frac{P_{Ne}}{P_{Ng}}$ ($\frac{N_e}{N_i}$)

≥ 1.

≥ 0 dB.

这里 N_a 是加性噪声. N_e 是输入端的加性噪声. $N_e\cdot K_p = N_a$.

同样 N_i 即为 N_g 定义为信号源内阻的输出最大功率. $\to kTB$.

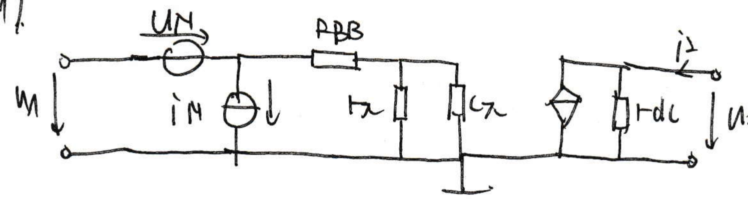
对于无源电阻.

$$NF = \frac{N_o}{K_{pm}\cdot N_i} = \frac{kTB}{K_{pm}\cdot kTB} = \frac{1}{K_{pm}} = L.$$

对于[illegible]放大. 噪声系数定义为 F. $NF = F$ dB

RA-01.

1).



可以用电压源和电流源代替噪声
若输入电阻Rg无穷大 → UN无效，IN有效
短路 → UN有效，IN无效
之所以没有第两次，是因为他们也有相关性。
计算中可以像前面一样打分为1.

2) $\begin{pmatrix} S_{UN} & , S_{UI} \\ S_{IN} & , S_{IN} \end{pmatrix} = \begin{pmatrix} G_{11} & G_{12} & G_{13} \\ G_{21} & G_{22} & G_{23} \end{pmatrix} \begin{pmatrix} S_{UNBB} & 0 & 0 \\ 0 & S_{INB} & 0 \\ 0 & 0 & S_{INC} \end{pmatrix} \begin{pmatrix} G_{11}^* & G_{21}^* \\ G_{12}^* & G_{22}^* \\ G_{13}^* & G_{23}^* \end{pmatrix}$

$= \begin{pmatrix} G_{11} S_{UNBB} & G_{12} S_{INB} & G_{13} S_{INC} \\ G_{21} S_{UNBB} & G_{22} S_{INB} & G_{23} S_{INC} \end{pmatrix} \begin{pmatrix} G_{11}^* & G_{21}^* \\ G_{12}^* & G_{22}^* \\ G_{13}^* & G_{23}^* \end{pmatrix}$

因 $G_{1.1}$, $G_{1.2}$, $G_{1.3}$ 已经求过，Volterra的分析，注意一下方向和Volterra相反.

$\begin{cases} G_{11} = -1, \\ G_{12} = R_{BB} \\ G_{13} = -\left[\frac{1}{g_m} + \frac{R_{BB}}{b}(1+j\omega C_x)\right] \end{cases}$ $\begin{cases} G_{21} = 0 \\ G_{22} = 1 \\ G_{23} = \frac{1+j\omega C_x}{b} \end{cases}$

因① 开路 → [illegible]，$i_1 = 0$，开路. U_{BB} 不影响电流.

因② i_N 和 i_{NB} 对电流无影响，由于 i_{NB} 是 r_x 的受控源，所以 i_N 和 i_{NB} 都是流过 $r_x // j\omega C_x$ 的电流

因③ $U_x = \frac{-i_{NC}}{g_m} = -i_N \cdot \frac{r_x}{1+r_x j\omega C_x}$

$\frac{i_N}{i_{NC}} = \frac{1+r_x j\omega C_x}{r_x \cdot g_m} = \frac{1+j\omega C_x \cdot r_x}{b}$

3). $S_{UN} = |G_{11}|^2 \cdot S_{UBB} + |G_{12}|^2 \cdot S_{INB} + |G_{13}|^2 \cdot S_{INC} = 2.17 \times 10^{-18}\ V^2/Hz$

$= |G_{11}|^2 \cdot 4kTR_{BB} + |G_{12}|^2 \cdot 4kT \cdot \frac{g_m}{2b} + |G_{13}|^2 \cdot 4kT \cdot \frac{g_m}{2}$

$S_{IN} = |G_{21}|^2 \cdot 4kTR_{BB} + |G_{22}|^2 \cdot 4kT \cdot \frac{g_m}{2b} + |G_{23}|^2 \cdot 4kT \cdot \frac{g_m}{2} = 7.3 \times 10^{-24}\ A^2/Hz$

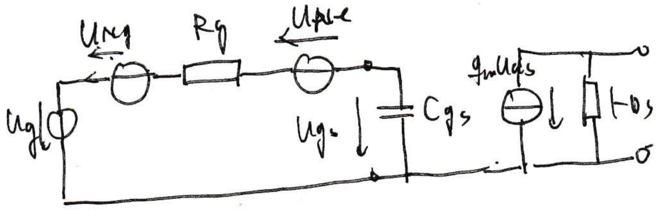
$S_{UI} =$ ~~$|G_{11} G_{21}^*| \cdot S_{UBB} + |G_{12} G_{2}|$~~

$G_{11} \cdot G_{21}^* S_{UBB} + G_{12} \cdot G_{21}^* S_{INB} + G_{13} \cdot G_{23}^* S_{INC} = (-0.873 - j \cdot 12.6) \times 10^{-21}\ W/Hz$

$S_{IUN} = S_{UIN}^*$

RA-02.

1).



2) ~~$U_{gs} | U_{Ng} = \frac{\frac{1}{j\omega C_{gs}} \cdot U_{Ng}}{R_g + \frac{1}{j\omega C_{gs}}} = \frac{U_{Ng}}{1+R_g j\omega C_{gs}} = U_{Ne}$~~

$\underline{S}_{QD} = \underline{c}\eta \cdot \sqrt{S_G \cdot S_D}$

我们要将相关变量转化为不相关变量.

~~$G_{12} = j\omega C_{gs}$ $\quad G_{13} = \frac{1}{g_m}$~~

$G_{11} = \frac{-R_g}{j\omega C_{gs}}$ $\quad G_{13} = \frac{1+j\omega C_{gs} \cdot R_g}{g_m}$

$$\begin{pmatrix} S_{UN} & \underline{S}_{UI} \\ \underline{S}_{IU} & S_{IN} \end{pmatrix} = \begin{pmatrix} \underline{G}_{11} & \underline{G}_{12} \\ \underline{G}_{21} & \underline{G}_{22} \end{pmatrix} \cdot \begin{pmatrix} S_{ING} & \underline{S}_{QD} \\ \underline{S}_{QD}^* & S_{IND} \end{pmatrix} \cdot \begin{pmatrix} \underline{G}_{11}^* & \underline{G}_{21}^* \\ \underline{G}_{12}^* & \underline{G}_{22}^* \end{pmatrix}$$

~~S_{UN}~~
$$= \begin{pmatrix} G_{11} \cdot S_{ING} + G_{12} \cdot S_{QD}^* & , \ \underline{G}_{11} \cdot \underline{S}_{QD} + \underline{G}_{12} \cdot S_{IND} \\ \ldots & , \ \ldots \end{pmatrix} \cdot \begin{pmatrix} G_{11}^* & G_{21}^* \\ G_{12}^* & G_{22}^* \end{pmatrix}$$

$$= |\underline{G}_{11}|^2 \cdot S_{ING} + 2 \cdot [G_{11}^* G_{12} \cdot S_{QD}^*]_{Re} + |G_{12}|^2 \cdot S_{IND}.$$

$$= \left|\frac{1}{g_m}\right|^2 \cdot$$

这里推导一下. G_{12} 表示和 Vorlesung 可以相同

$U_{gs} = \frac{i_N}{g_m} = U_{ne} \cdot \frac{\frac{1}{j\omega C_{gs}}}{R_g + \frac{1}{j\omega C_{gs}}} \rightarrow \frac{U_{ne}}{i_N} = \frac{1}{g_m} \cdot (1 + R_g \cdot j\omega C_{gs})$

3) 代值求解. $1.26 \times 10^{-18} \frac{V^2}{Hz}$.

4). $F = 1 + \frac{S_{une}}{S_{ung}} = 2.575$

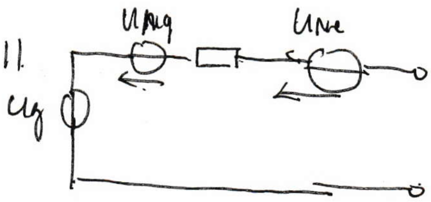
$NF = 10 \lg F = 4.11 dB$

5) $SNR_{dB} = 10 \lg SNR$, $\quad SNR = \frac{\nu P_g}{\nu (P_g + P_{Ng})} = \frac{\frac{\hat{u}_g^2}{2} / R}{\frac{B \cdot (S_{Ung} + S_{Une})}{R}} = 48543.7$.

$\Rightarrow SNR_{dB} = 46.9 dB$ 　有效值の平方.

RA-03.

1).



(2) $\underline{S}_{UNe} = (\underline{G}_1, \underline{G}_2) \begin{pmatrix} \underline{S}_{UN} & , \underline{S}_{UI} \\ \underline{S}_{IN} & , S_{IN} \end{pmatrix} \begin{pmatrix} G_1^* \\ G_2^* \end{pmatrix}$

$G_1 = -1 \quad G_2 = -R_g. \rightarrow \underline{S}_{UNe} = \underline{S}_{UN} + R_g^2 \cdot S_{IN} + 2R_g \cdot Re\{S_{UI}\}$
$= 2.3 \times 10^{-18} \frac{V^2}{Hz}$.

3). $F = 1 + \frac{S_{une}}{S_{ung}} = 3.875$. $\quad NF = 5.88 dB$.

4). $SNR_{dB} = \frac{\frac{1}{2}|\hat{u}_g|^2}{B(S_{ung} + S_{Une})} = 55.1 dB$.