4. Parameter der Nichtlinearität.

4.1. Modell für die Nichtlinearität.

$$U_a = f(U_g) = f(U_{gA} + \cancel{} u_g)$$

$$U_a = U_{aA} + u_a = f(U_{gA}) + a_1 u_g + a_2 u_g^2 + a_3 u_g^3$$

- erwartetes Kleinesignalanalyse

泰勒公式.

$$U_a = f(U_g)$$

$$\Rightarrow u_a = \frac{df}{du_g}\Big|_A u_g + \frac{1}{2}\frac{d^2f}{du_g^2}\Big|_A \cdot u_g^2 + \frac{1}{6}\frac{d^3f}{du_g^3}\Big|_A \cdot u_g^3$$

$$\Rightarrow U_g = g(U_a)$$

$$\Rightarrow b_1, b_2, b_3 \ldots$$

基本共射电路

$$U_a = U_{cc} - R_c \cdot I_s \exp\left(\frac{U_{BEA} + u_g}{U_T}\right) \leftarrow \exp\frac{U_{BEA}}{U_T} \cdot \exp\frac{u_g}{U_T}$$

$$= U_{cc} - \left[R_c I_{CA} + \frac{R_c I_{CA}}{U_T} u_g + \frac{R_c I_{CA}}{2}\frac{u_g^2}{(U_T)^2} + \frac{R_c I_{CA}}{6}\frac{u_g^3}{U_T^3}\right]$$

$$= U_{cc} - I_{CA} R_c \left[1 + \frac{u_g}{U_T} + \frac{1}{2}\left(\frac{u_g}{U_T}\right)^2 + \frac{1}{6}\left(\frac{u_g}{U_T}\right)^3\right]$$

基本差分.差模传输特性.

$$g_m = \frac{I_{EQ}}{U_T} = \frac{I_0}{2U_T} \qquad I_0 = i_{E1} + i_{E2} = I_s e^{\frac{U_{BE1}}{U_T}} + I_s e^{\frac{U_{BE2}}{U_T}}$$

$$= i_{E2}\left(1 + e^{\frac{U_{BE1} - U_{BE2}}{U_T}}\right) = i_{E1}\left(1 + e^{\frac{U_{BE2} - U_{BE1}}{U_T}}\right)$$

$$i_{C1} - i_{C2} = I_0\left[\frac{1}{1 + e^{-\frac{U_{diff}}{U_T}}} - \frac{1}{1 + e^{\frac{U_{diff}}{U_T}}}\right]$$

$$= I_0\left[\frac{1 + e^{\frac{u_x}{U_T}} - 1 - e^{-\frac{u_x}{U_T}}}{\left(1 + e^{-\frac{u_x}{U_T}}\right)\left(1 + e^{\frac{u_x}{U_T}}\right)}\right] = I_0 \cdot \frac{e^{\frac{u_x}{U_T}} - e^{-\frac{u_x}{U_T}}}{1 + e^{\frac{u_x}{U_T}} + e^{-\frac{u_x}{U_T}} + 1}$$

$$= I_0\left[\frac{e^{\frac{u_x}{2U_T}} \cdot e^{\frac{u_x}{2U_T}} - e^{-\frac{u_x}{2U_T}} \cdot e^{-\frac{u_x}{2U_T}}}{e^{\frac{u_x}{2U_T}} \cdot e^{-\frac{u_x}{2U_T}} + e^{\frac{u_x}{2U_T}} \cdot e^{\frac{u_x}{2U_T}} + e^{-\frac{u_x}{2U_T}} e^{-\frac{u_x}{2U_T}} + e^{-\frac{u_x}{2U_T}} e^{+\frac{u_x}{2U_T}}}\right]$$

$$= I_0 \frac{\left(e^{\frac{u_x}{2U_T}} + e^{-\frac{u_x}{2U_T}}\right)\left(e^{\frac{u_x}{2U_T}} - e^{-\frac{u_x}{2U_T}}\right)}{\left(e^{\frac{u_x}{2U_T}} + e^{-\frac{u_x}{2U_T}}\right)^2} = I_0 \frac{e^{\frac{u_x}{2U_T}} - e^{-\frac{u_x}{2U_T}}}{e^{\frac{u_x}{2U_T}} + e^{-\frac{u_x}{2U_T}}} = I_0 \,\mathrm{th}\frac{u_x}{U_T}$$

$$U_a = U_{cc} - \frac{R_c I_0}{2}\left[\frac{u_g}{2U_T} - \frac{1}{3}\left(\frac{u_g}{2U_T}\right)^3\right]$$

4.2. 傅里叶.

$$u_g = \hat{U}_g \cos(w_0 t), \Rightarrow u_a = a_1 \hat{u}\cos(w_0 t) + a_2 \hat{u}^2 \cos^2(w_0 t) + a_3 \hat{u}^3 \cos^3(w_0 t)$$

$$= a_1 \hat{u}\cos(w_0 t) + a_2 \hat{u}^2 \frac{\cos(2w_0 t) + 1}{2} + a_3 \hat{u}^3\left(\frac{3}{4}\cos w_0 t + \frac{1}{4}\cos 3w_0 t\right)$$

$$= \frac{1}{2} a_2 \hat{u}_g^2 + \left(a_1 + \frac{3}{4} a_3 \hat{u}_g^2\right)\hat{u}_g \cos w_0 t + \frac{a_2 \hat{u}_g^2}{2}\cos(2w_0 t) + \frac{a_3}{4}\hat{u}_g^3 \cos(3w_0 t)$$

谐波失真.

$$HD_2 = \left|\frac{A_2}{A_1}\right| = \frac{1}{2} a_2 \hat{u}_g^2 \Big/ \left(a_1 + \frac{3}{4} a_3 \hat{u}_g^2\right)\hat{u}_g \quad \text{对于 } a_1 \gg \frac{3}{4} a_3 \hat{u}_g^2, \; = \frac{1}{2}\left|\frac{a_2}{a_1}\right| \hat{u}_g$$

$$HD_3 = \left|\frac{A_3}{A_1}\right| = \frac{1}{4} a_3 \hat{u}_g^3 \Big/ \left(a_1 + \frac{3}{4} a_3 \hat{u}_g^2\right)\hat{u}_g, \; \ldots\ldots, \; = \frac{1}{4}\left|\frac{a_3}{a_1}\right| \hat{u}_g^2$$

非线性谐波失真系数.

$$K=\sqrt{\frac{\sum_{n=0}^{\infty} A_n^2}{\sum_{n=1}^{\infty} A_n^2}}$$

总谐波失真

$$THD=\frac{\sqrt{\sum_{n=2}^{\infty} A_n^2}}{|A_1|}=\sqrt{\frac{\sum_{n=2}^{\infty}\frac{1}{T}\int_0^T (A_n\cos n\omega_0 t)^2\,dt}{\frac{1}{T}\int_0^T (A_1\cos\omega_0 t)^2\,dt}}$$

1dB 压缩点

$$20\lg\frac{Gain-Linear\cdot P_{in}}{Gain-notLinear\cdot P_{in}}=1dB.$$

线性增益 $\frac{A_1}{\hat{u}_g}=a_1+\frac{3}{4}a_3\hat{u}_g^2=V_B.$

在这里 $20\lg\left(\frac{V_B}{a_1}\right)=20\lg\left(1+\frac{3}{4}\frac{a_3}{a_1}\hat{u}_g^2\right)=\pm 1dB.$

$$\hat{u}_{g\,\pm 1dB}=\sqrt{\frac{4a_1}{3a_3}\left(10^{\pm\frac{1}{20}}-1\right)}$$

三阶截止点

$$\left|\frac{A_2}{A_1}\right|=\left|\frac{A_3}{A_1}\right|=1.$$

$$HD_2=\left|\frac{A_2}{A_1}\right|$$

$$HD_3=\left|\frac{A_3}{A_1}\right|.$$

$$1/P_{HD2}=\left|\frac{A_2}{A_1}\right|=\frac{\frac{1}{2}a_2\hat{u}_g^2}{a_1\hat{u}_g+\frac{3}{4}a_3\hat{u}_g^3}=\frac{\frac{1}{2}a_2\hat{u}_g}{a_1+\frac{3}{4}a_3\hat{u}_g^2}\approx\frac{1}{2}\frac{a_2}{a_1}\hat{u}_g=1\Rightarrow\hat{u}_g=2\left|\frac{a_1}{a_2}\right|$$

$$1/P_{HD3}=\left|\frac{A_3}{A_1}\right|\approx\frac{1}{4}\frac{a_3}{a_1}\hat{u}_g^2=1.\Rightarrow\hat{u}_g=2\sqrt{\frac{a_3}{a_1}}.$$

NL-01.

不考虑厄立效应($U_Y \gg U_{CB}$)

1. $U = I \cdot R + U_{BE}$

$= I \cdot R + U_T \ln \frac{I}{I_S}$

$U_0 = 50\times10^{-6}\times10^{3} + 26\times10^{-3} \ln \frac{50\times10^{-6}}{4.75\times10^{-15}}$

$= 650\,mV$.

2. $U = IR \cdot U_T \ln \frac{I}{I_S}$

$a_0 = U_0 = 650\,mV$.

$a_1 = \frac{dU}{dI}\Big|_{I=I_0} = R + U_T \frac{I_S}{I_0} \cdot \frac{1}{I_S} = R + \frac{U_T}{I_0} = 1520\,\frac{V}{A}$.

$a_2 = \frac{1}{2}\frac{d^2U}{dI^2}\Big|_{I=I_0} = \frac{-U_T}{2 I_0^2} = -5.2\times10^{6}\,V/A^2$

$a_3 = \frac{1}{6}\frac{d^3U}{dI^3}\Big|_{I=I_0} = \frac{1}{6} \cdot U_T \cdot -2 \cdot \frac{1}{I_0^3} = \frac{U_T}{3 I_0^3} = 69.3\cdot10^{9}\,V/A^3$

3. $U = a_0 + a_1 \cdot \hat{I}\cos\omega t + a_2 (\hat{I}\cos\omega t)^2 + a_3 \cdot (\hat{I}\cos\omega t)^3$

$A_0 = a_0 + \frac{a_2}{2}\hat{I}^2 = 649\,mV$

$A_1 = (a_1 \hat{I} + \frac{3}{4} a_3 \hat{I}^3) = 30.8\,mV$

$A_2 = \frac{a_2}{2}\hat{I}^2 = 1.09\,mV$

$A_3 = \frac{1}{4} a_3 \hat{I}^3 = 0.139\,mV$.

4. $HD_2 = \left|\frac{A_2}{A_1}\right| = 0.0338$ $\quad HD_3 = \left|\frac{A_3}{A_1}\right| = 0.0045$.

5. $\cancel{I_{1dB}=}$ $V_B = \frac{A_1}{\hat{I}} = a_1 + \frac{3}{4} a_3 \hat{I}^2$. $\quad 20\lg\frac{V_B}{a_1} = \pm 1\,dB$.

$a_1 = R + \frac{U_T}{I_0}$ $\quad I_{\pm 1dB} = \sqrt{\frac{4a_1}{3a_3}(10^{\frac{\pm 1}{20}} - 1)}$ ，由题目，取正

$I_{1dB} = \sqrt{\frac{4a_1}{3a_3}(10^{\frac{1}{20}} - 1)} = 5.97\times10^{-5}\,A = 59.73\,\mu A$

6. $IIP\,HD_2 \approx 2\left|\frac{a_1}{a_2}\right| = \cancel{58}\ 584.6\,\mu A$.

$IIP\,HD_3$: $\left|\frac{A_3}{A_1}\right| = 1$ $\quad \frac{\frac{1}{4}a_3 \hat{I}^2}{a_1 + \frac{3}{4}a} \Rightarrow IIP\,H_3 = 2\sqrt{\left|\frac{a_1}{a_3}\right|} = 296.1\,\mu A$.

7. 设 $i = b_1 u + b_2 u^2 + b_3 u^3$

$= b_1(a_1 i + a_2 i^2 + a_3 i^3) + b_2(a_1 i + a_2 i^2 + a_3 i^3)^2 + b_3(a_1 i + a_2 i^2 + a_3 i^3)^3$

$\Rightarrow$

$i = b_1 a_1 i + (a_1^2 b_2 + b_1 a_2) i^2 + (2 b_2 a_1 a_2 + b_1 a_3 + b_3 a_1^3) i^3$.

$$\Rightarrow \begin{cases} b_1 a_1 = 1 & \Rightarrow b_1 = \frac{1}{a_1} = 0.658\times10^{-3}\,A/V \\ a_1^2 b_2 + b_1 a_2 = 0 & \Rightarrow b_2 = \frac{-a_2}{a_1^3} = 1.48\times10^{-3}\,A/V^2 \\ 2 b_2 a_1 a_2 + b_1 a_3 + b_3 a_1^3 = 0 & \Rightarrow b_3 = \frac{2\frac{a_2^2}{a_1} - a_3}{a_1^4} = -6.32\times10^{-3}\,A/V^3 \end{cases}$$

8. wie. a_1, a_2, a_3, a_0.

$$B_0 = U_0 + \frac{b_2}{2}\hat{U}^2$$

$$B_1 = b_1\hat{U} + \frac{3b_3}{4}\hat{U}^3 = 13.1\,mA$$

$$B_2 = \frac{b_2}{2}\hat{U}^2 = 292\,nA$$

$$B_3 = \frac{1}{4}b_3\hat{U}^3 = 12.6\,mA.$$

9. $HD_2 = \left|\frac{B_2}{B_1}\right| = 0.0226$ $\qquad HD_3 = \left|\frac{B_3}{B_1}\right| = 0.00f6t.$

NL-02.

1. ~~Ce~~ $\cancel{I_aA = I_0\tanh\frac{U_A}{2U_T}}$.

$I_aA = 0 \Rightarrow U_eA = 0$.

2. $I_a = \sqrt{\beta \cdot I_0} \cdot U_e \cdot \sqrt{1 - \frac{\beta}{4 \cdot I_0}U_e^2} = \sqrt{\beta I_0 U_e^2 - \frac{\beta^2}{4}U_e^4}$

$$a_1 = \frac{dI_a}{dU_e}\Big|_{U_e=0} = \sqrt{\beta \cdot I_0} \cdot \sqrt{1 - \frac{\beta}{4 I_0}U_e^2} + \sqrt{\beta \cdot I_0} \cdot U_e \cdot \ldots = \sqrt{\beta \cdot I_0}$$

$$a_2 = \frac{d^2 I_a}{2\,dU_e^2}\Big|_{U_e=0} = 0 \cdot \left[\frac{2\beta I_0 - \beta^2 U_e^2}{2\sqrt{\beta I_0 - \frac{\beta}{4}U_e^2}}\right]' = 0 \cdot \cancel{\frac{-4\beta^2 U_e\sqrt{\beta I_0 - \frac{\beta}{4}U_e} - (2\beta I_0 - \beta^2 U_e^2) \cdot \frac{-\frac{\beta}{2}U_e}{\sqrt{\beta I_0 - \frac{\beta}{4}U_e^2}}}{4(\beta I_0 - \frac{\beta}{4}U_e^2)}}$$

$$a_3 = \frac{d^3 I_a^3}{6\,dU_e^3}\Big|_{U_e=0} = 0 \cdot -\frac{1}{8}\sqrt{\frac{\beta^3}{I_0}}$$

3. $$HD_3 = \left|\frac{A_3}{A_1}\right| = \left|\frac{\frac{1}{4}a_3\hat{U}_e^2}{a_1 + \frac{3}{4}a_3}\right| \approx \left|\frac{1}{4}\frac{a_3}{a_1}\hat{U}_e^2\right| = \left|\frac{1}{4} \cdot -\frac{\beta}{8 I_0}\hat{U}_e^2\right| = \frac{\beta}{32 I_0}\hat{U}_e^2 = 0.01.$$

$$\Rightarrow \beta = \frac{0.32}{U_e^2} I_0 \qquad g_m = \sqrt{\beta I_0} \Rightarrow I_0 = \frac{g_m^2}{\beta}$$

$$\beta = \sqrt{\frac{32 HD_3 g_m^2}{\hat{U}_e^2}} \qquad \frac{W}{L} = \beta / kp = 5f6. \qquad I_0 = \frac{g_m^2}{\beta} = 442\,mA.$$

NL-03.

1. $U_eA = U_{RE1} + R_E \cdot \frac{I_a}{2} - U_{BE1}$.

$I_aA = 0A \Rightarrow U_eA = 0V$.

2. $g'_T = \frac{dI_a}{dU_e}\Big|_{U_e=0}$, $\quad U_e = I_a \cdot \frac{R_e}{2} + 2U_T \operatorname{arctanh}\left(\frac{I_a}{I_0}\right)$

$$\frac{dU_e}{dI_a}\Big|_{I_a=0} = \left[\frac{R_e}{2} + \frac{2U_T}{I_0}\frac{1}{1 - \left|\frac{I_a}{I_0}\right|^2}\right]\Big|_{I_a=0} = \frac{R_e}{2} + \frac{2U_T}{I_0}$$

$$g'_T = \frac{1}{\frac{R_e}{2} + \frac{2U_T}{I_0}}$$

3. $$HD_3 = \frac{1}{48}\left(\frac{g_T'}{g_T}\right)^3\left(\frac{\hat{U}_e}{U_T}\right)^2,$$

$$b_1 = \frac{1}{a_1}, \quad b_2 = \frac{-a_2}{a_1^3}, \quad b_3 = \frac{2\frac{a_2}{a_1} - a_3}{a_1^4}$$

$$HD_3 = \left|\frac{\frac{1}{4}b_3\hat{I}^2}{b_1 + \frac{3}{4}b_3\hat{I}^2}\right|$$