14. Erzeugung orthogonaler Signale.

通信之道、信号调制的简单说明（浅析）

双边带信号调制后负半轴会占用频谱，所以我们可以用希尔伯特变换得到单边谱信号

$S_{PF+}(t) = [s(t) + j\hat{s}(t)]\cos(w_c t)$

希尔伯特变换是非因果变换，对于模拟电路来说实现难度较大，所以通常不这样做。

IQ调制则是利用了 $\cos w t$，$\sin w t$ 相互正交和频率相同，所以可以同时利用相位和频率将带宽提升两倍。

我们可以证明，用复指数调制后取实部，则可以得到信号

~~R Ex~~ $[x(t) + j y(t)] e^{jwt}$，取实部 $\longrightarrow$ $x(t)\cos wt - y \sin wt$.

我们还可以证明，任意一个实信号都可以由复信号和复指数调制，也就是说，IQ调制可以用来调制实信号，而我们可以用复信道来等价表达。

所以我们的重点则是12.1 [AS] 就是得到 I, Q 信号。

先数学推导一下，$\Rightarrow$ 任意一个实信号 $S_{PF}(t)$ 都可以写成 $S_{PF}(t) = R[s(t) e^{jw_c t}]$，$s(t) = x(t) + j y(t)$

将

$S_{PF}(t) = R[s(t) e^{jw_c t}] = \frac{1}{2}[s(t) e^{jw_c t} + s^*(t) e^{-jw_c t}]$ 做傅里叶变换

$S_{PF}(w) = \frac{1}{2}[S(w - w_c) + S^*(-w - w_c)]$.

其中 $S_{PF}(w) = F[S_{PF}(t)]$，$F(s(t) e^{jw_c t}) = S(w - w_c)$

$F[s^*(t)] = S^*(-w)$

$S^*_{PF}(-w) = S_{PF}(w)$（因为是实信号），所以

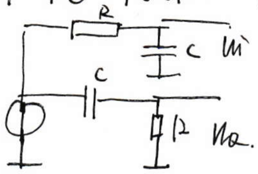
$$\Rightarrow \quad S(w - w_c) = \begin{cases} 0 & w < 0 \\ S_{PF}(w) & w = 0 \\ 2S_{PF}(w) & w > 0 \end{cases} \qquad S^*(-w - w_c) = \begin{cases} 2S_{PF}(w) & w < 0 \\ S_{PF}(w) & w = 0 \\ 0 & w > 0 \end{cases} \Rightarrow S(w) = \begin{cases} 0 & w < -w_c \\ S_{PF}(w + w_c) & w = -w_c \\ 2S_{PF}(w + 2w_c) & w > -w_c \end{cases}$$

就是把负半轴切掉，再向左移 w_c，就可以了。

如果我们将信道也按这样的变换的话，可以得到 $R_{PF}(w) = \frac{1}{2}[R(w - w_c) + R^*(-w - w_c)]$，$R(w) = \frac{1}{2} S(w) \cdot H(w)$

实际电路，复信号也是与实信号等价。

1. RC-Netzwerke.



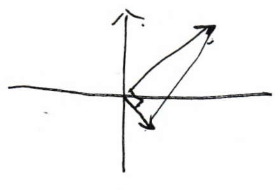
$$\frac{U_i}{U_e} = \frac{1}{1 + jwRC} = \frac{1}{\sqrt{1 + w^2R^2C^2}}(1 - jwRC) = \frac{1}{\sqrt{1 + w^2R^2C^2}} \exp[\arctan(-wRC)]$$

$$\frac{U_a}{U_e} = \frac{jwRC}{1 + jwRC} = \frac{1}{\sqrt{1 + w^2R^2C^2}}(w^2R^2C^2 + jwRC) = \frac{wRC}{\sqrt{1 + w^2R^2C^2}} \exp[\arctan(\frac{1}{wRC})]$$

$\phi_{ui} = \arctan(-wRC)$ $\qquad \phi_{ua} = \frac{\pi}{2} + \arctan(-wRC)$ $\qquad \phi_{ua} - \phi_{ui} = \frac{\pi}{2}$

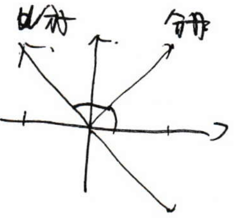
若元件有误差 $\phi = \frac{\pi}{2} - \arctan(-wRC) + \arctan(w(1+\alpha)R \cdot (1+\beta)C)$

若 $\alpha \ll 1$, $\beta \ll 1$ $\quad \Delta\phi = \phi - \frac{\pi}{2} = \arctan(\frac{\alpha + \beta}{2})$，对于 $wRC = 1$.



若将上面的 U_i 和 U_a 作差，记为 U_Q，那么相差则和上面的情况不一样，如图。

$\frac{U_a}{U_e} = \frac{1 - jwRC}{1 + jwRC}$ — 模值为 1，角度为 ~~arctan(-wRC) − arctan(wRC)~~ $\pi - 2\arctan(wRC)$



数字举例. $C = 100\,pF$ $\quad R = 2.27\,k\Omega$，$f_g = 700\,MHz$

$wRC = 0.998$ $\qquad \phi \approx 90°$ 这种相位会受频率影响，上一个理想情况下则没有。

$\pi - 2\arctan(2\pi f_{H,L} \cdot RC) = 90° \pm 2°$

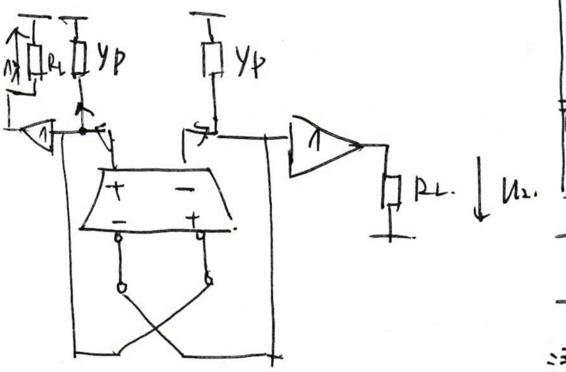
$\Rightarrow \arctan(2\pi f_{H,L} \cdot RC) = \frac{\mp 2° + 90°}{2} = \mp 1° + 45°$

$f_H = 726\,MHz$. $\qquad B_{f,20} = 49\,MHz$.

$f_L = 677\,MHz$ $\qquad f_H / f_L = \tan 46° / \tan 44° = 1.07$.

RC-Netzwerk nach WEAVER. (Übung 中不涉及此项).
基本思路就是用基尔霍夫电压定律和电流定律

跨导振荡器.



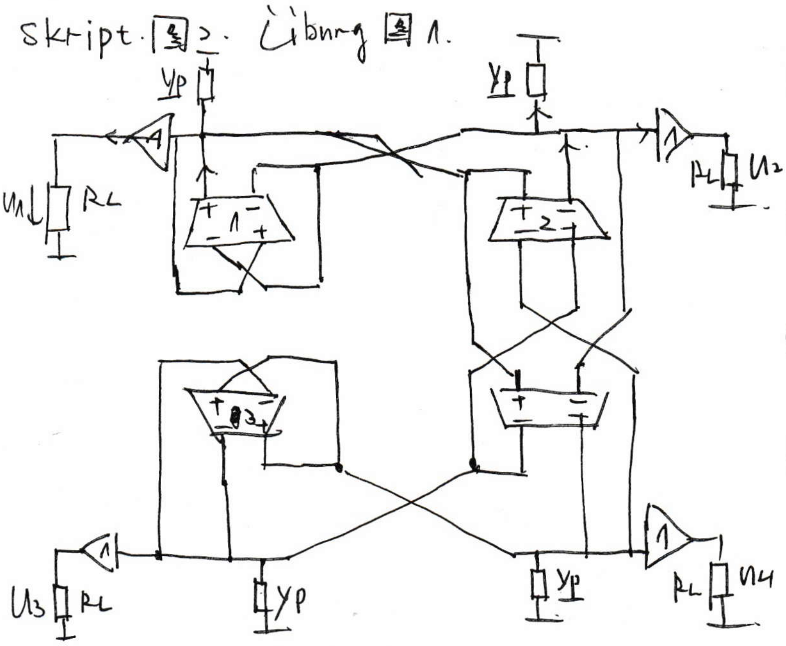
Buffer 即为缓冲器的作用:
跨导放大器直接接负载则会影响放大倍数.
而加一个共栅或共漏电路就可以利用其较高的输入电阻提高放大倍数. 同时其输出端输出电阻很低. 可以使负载分得较高的电压.

$-u_1 = \frac{1}{y_p}\cdot g_m(u_2-u_1) \Rightarrow u_1 = -u_2$, 代入后

$-u_2 = \frac{1}{y_p}\cdot g_m(u_1-u_2) \Rightarrow \frac{1}{y_p}\cdot g_m = \frac{1}{2} \Rightarrow g_m = \frac{y_p}{2}$

注意, 输出端(OTA) "+" 向外为正. "−" 向内为正.

Skript 图2. Übung 图1.



对每个节点列方程

$g_{T1}:\begin{cases} u_{1,1} = \frac{g_{T1}}{y_p}(u_1-u_2) \\ u_{2,1} = \frac{g_{T1}}{y_p}(u_2-u_1) \end{cases}$

$g_{T2}:\begin{cases} u_{2,2} = \frac{g_{T2}}{y_p}(u_4-u_3) \\ u_{1,2} = \frac{g_{T2}}{y_p}(u_3-u_4) \end{cases}$

$g_{T3}:\begin{cases} u_{4,3} = \frac{g_{T3}}{y_p}(u_4-u_3) \\ u_{3,3} = \frac{g_{T3}}{y_p}(u_3-u_4) \end{cases}$

$g_{T4}:\begin{cases} u_{4,4} = \frac{g_{T4}}{y_p}(u_1-u_2) \\ u_{3,4} = \frac{g_{T4}}{y_p}(u_2-u_1) \end{cases}$

$g_T = g_{T1} = g_{T3}, \quad g_{Tc} = g_{T2} = g_{T4}.$

$u_1 - u_2 = u_i \qquad u_3 - u_4 = u_Q$

~~从 $g_{T1}, g_{T3} \Rightarrow \begin{cases} u_1 = -u_2 \\ u_3 = -u_4 \\ g_T = \frac{y_p}{2} \end{cases}$ 代入 g_{T2}, g_{T4} 即 g_{Tc}, $\begin{cases} u_2 = \frac{g_{Tc}}{y_p}\cdot 2u_4 = -2u_4 \\ u_1 = \frac{g_{Tc}}{y_p}\cdot -2u_4 = 2u_3 \end{cases}$ $u_4 = \frac{g_{Tc}}{y_p}2u_1 = -\frac{g_{Tc}}{y_p}\cdot 2u_2$, $u_3 = \frac{g_{Tc}}{y_p}2u_2 = -\frac{g_{Tc}}{y_p}\cdot 2u_1$~~

~~$u_i = u_1 - u_3 = \frac{g_{Tc}}{y_p}\cdot 2u_3 - u_3$~~

从跨导部分会存在电流方程的平衡问题, 故从 u、y_p 出发

(1) $u_1 = u_{1,1} + u_{1,2} = \frac{1}{y_p}[g_T(u_1-u_2) + g_{Tc}(u_3-u_4)]$

(2) $u_2 = u_{2,1} + u_{2,2} = \frac{1}{y_p}[g_T(u_2-u_1) + g_{Tc}(u_4-u_3)]$

(3) $u_3 = u_{3,3} + u_{3,4} = \frac{1}{y_p}[g_T(u_3-u_4) + g_{Tc}(u_2-u_1)]$

(4) $u_4 = u_{4,3} + u_{4,4} = \frac{1}{y_p}[g_T(u_4-u_3) + g_{Tc}(u_1-u_2)]$.

①+②, ③+④:
$u_1 = -u_2$
$u_3 = -u_4$

①−②, ③−④:
$u_i = \frac{2g_T}{y_p}u_i + \frac{2g_{Tc}}{y_p}u_Q$
$u_Q = \frac{2g_T}{y_p}u_Q - \frac{2g_{Tc}}{y_p}u_i$

$u_i = \left(\frac{2g_{Tc}}{y_p}\Big/1-\frac{2g_T}{y_p}\right)u_Q$

$u_Q = \left(\frac{-2g_{Tc}}{y_p}\Big/1-\frac{2g_T}{y_p}\right)u_i$

$\Rightarrow -\left(\frac{2g_{Tc}}{y_p}\Big/1-\frac{2g_T}{y_p}\right)^2 = 1 \Rightarrow \begin{matrix} u_i = j u_Q \\ u_Q = -j u_i \end{matrix}$

$\frac{2g_{Tc}}{y_p}\Big/1-\frac{2g_T}{y_p} = \pm j$

$\frac{\frac{2g_{Tc}}{y_p}}{1-\frac{2g_T}{y_p}} = \pm j \Rightarrow \frac{2g_{Tc}}{y_p - 2g_T} = \pm j$, $y_p = g_p + j\omega C_p + \frac{1}{j\omega L_p}$, $y_p = 2g_T \pm 2j g_{Tc} \Rightarrow \begin{cases} g_T = \frac{g_p}{2} \\ \omega \text{ 如下式} \end{cases}$

$\omega C_p - \frac{1}{\omega L_p} = \pm 2g_{Tc}$, $\omega^2 L_p C_p - 1 = \pm 2g_{Tc}\cdot\omega L_p$, $L_pC_p\omega^2 \mp 2g_{Tc}L_p\cdot\omega - 1 = 0 \Rightarrow \omega = (\pm)\frac{g_{Tc}}{C_p} \pm \sqrt{\left(\frac{g_{Tc}}{C_p}\right)^2 + \frac{1}{C_pL_p}}$

$\approx \sqrt{\frac{1}{L_pC_p}} \pm \frac{g_{Tc}}{C_p}$