10. Prozessvariation.

绝对误差. $\Delta p = p_k - p_z$ ，相对误差 $\frac{\Delta p}{p} = \frac{p_k - p_z}{p_z}$ 目标. t_{ox}, W, L.

$$R = \frac{\rho L}{A} = \frac{\rho \cdot L}{W \cdot d} = \frac{\rho}{d} \cdot \frac{L}{W} = R_s \cdot \frac{L}{W}.$$

如果 $R = R_1 + R_2 + R_3$ ，讨论下误差情况

$$\Rightarrow \frac{\Delta R}{R} = \frac{\Delta R_1}{R_1} \cdot \frac{R_1}{R} + \frac{\Delta R_2}{R_2} \cdot \frac{R_2}{R} + \frac{\Delta R_3}{R_3} \cdot \frac{R_3}{R}$$ ，使 $\frac{\Delta R}{R} = y$. $\frac{R_i}{R} = a_i$ ， $\frac{\Delta R_i}{R} = x_i$

$\sigma_i = \left(\frac{\Delta R_i}{R_i}\right)$

目标： $\sigma\left(\frac{\Delta R}{R}\right) \rightarrow \min$. 确定系数 a_i，

$$\sigma^2 y = J \begin{bmatrix} \sigma^2(x_1) & \cdots & \sigma(x_1, x_n) \\ \vdots & \ddots & \vdots \\ \sigma^2(x_n, x_1) & \cdots & \sigma(x_n^2) \end{bmatrix} J^T$$

$$\sigma^2 ges = a_1^2 \sigma_1^2 + a_2 \sigma_2^2 + a_3 \sigma_3^2$$

mit $a_3 = 1 - a_2 - a_3$. $J = \left[\frac{\partial y}{\partial x_1}, \ldots, \frac{\partial y}{\partial x_n}\right]$.

极值点梯度为0. 这里是拉格朗日乘数来求△简化. $\Rightarrow$ 直接代入.

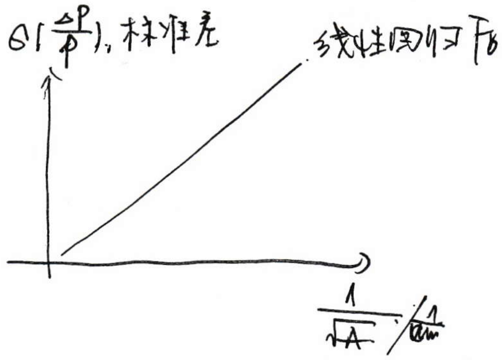
$$\frac{\partial \sigma^2 ges}{\partial a_1} = 2a_1\sigma_1^2 - 2(1-a_1-a_2)\sigma_3^2 = 0$$
$$\frac{\partial \sigma^2 ges}{\partial a_2} = 2a_2\sigma_2^2 - 2(1-a_1-a_2)\sigma_3^2 = 0.$$
$$\Rightarrow a_1 = \frac{1}{\sigma_1^2\left(\frac{1}{\sigma_2^2} + \frac{1}{\sigma_3^2}\right) + 1}$$
$$a_2 = \frac{1}{\sigma_2^2\left(\frac{1}{\sigma_1^2} + \frac{1}{\sigma_3^2}\right) + 1}$$

10.1. Motivation: 减小误差

10.2. Global variationen（全局）

10.2.1. 层级电阻计算. 串并联+电阻公式

10.2.2. 最小均方误差计算不相干电阻串联(3部分)



10.3. Lokale Variationen - 相邻器件

$\sigma(\Delta p) = \frac{A_p}{\sqrt{A}}$

$\sigma\left(\frac{\Delta p}{p}\right) = \frac{A_p}{\sqrt{A}}$

例. 1). 2个电阻. $R = 1k\Omega$，Fehler $< 0.2\%$. $A_R = 0.03\,\mu m$.

Ausbeute: 99.7%

$\frac{\Delta R}{R} = 0.2\% = 3 \cdot \sigma\left(\frac{\Delta R}{R}\right) \Rightarrow \frac{\Delta R}{R} = 3 \cdot \frac{A_R}{\sqrt{A}}$ $A = 2025\,\mu m$, in Zahlenbeispiel.

2) NMOS-Differentzpaar.

$I_{DA} = \frac{\beta}{2}(U_{GSA} - U_{th})^2$ ， $\sigma^2(\Delta I_{DA}) = \sigma^2(\Delta U_{th}) J^2_{Uth} + \sigma^2(\Delta\beta) \cdot J_\beta^2$.

$J_{Uth} = \frac{\partial I_{DA}}{\partial U_{th}} = -\beta(U_{GSA} - U_{th}) = -\beta \cdot \sqrt{\frac{-2I_{DA}}{\beta}} = \sqrt{2 I_{DA} \cdot \beta}$

$J_\beta = \frac{(U_{GSA} - U_{th})^2}{2} = \frac{I_{DA}}{\beta}$.

$$\Rightarrow \sigma^2\left(\frac{\Delta I_{DA}}{I_{DA}}\right) = \frac{1}{I_{DA}^2} \cdot J_\beta^2 \cdot \sigma^2(\Delta\beta) + \frac{1}{I_{DA}^2} \cdot J^2_{Uth} \cdot \sigma^2(\Delta U_{th})$$
$$= \sigma^2\left(\frac{\Delta\beta}{\beta}\right) + \frac{1}{I_{DA}^2} \cdot J^2_{Uth}\, \sigma^2(\Delta U_{th})$$
$$= \sigma^2\left(\frac{\Delta\beta}{\beta}\right) + \frac{2\beta}{I_{DA}} \cdot \sigma^2(U_{th}),$$

$\sigma^2\left(\frac{\Delta\beta}{\beta}\right) = \frac{A_\beta}{\sqrt{A}}$， $\sigma^2(U_{th}) = \frac{A_{Uth}}{\sqrt{A}}$， $2\beta = \frac{4}{(U_{GSA} - U_{th})^2}$

■ PV-01

$$\frac{\Delta R}{R} = 0.01 = 3\frac{A_R}{\sqrt{A}} \quad \Rightarrow A = \cancel{0.01}\left(\frac{3A_R}{0.01}\right)^2 = 728\ \mu m^2 \qquad \boxed{R = \rho\cdot\frac{L}{W} = \rho\cdot\frac{L^2}{A}},\ \rho = \frac{R_1\cdot A}{\cancel{A_R} L_1}$$

$$\cancel{K_L = \frac{R\cdot A}{\rho} = \sqrt{\frac{R\cdot A\cdot A}{R_A}}} \qquad \boxed{K_L = \sqrt{\frac{R_S\cdot A_S}{R_1\cdot A_R}} =}$$

$$\frac{R_S}{R_1} = \frac{\rho\cdot\frac{W}{L}}{\rho L_1(?)} = \frac{W}{L} \Rightarrow A = W\cdot L = \frac{R_S}{R_1}\cdot L^2 \Rightarrow L = 180.8\ \mu m. \quad W = 3.82\ \mu m.$$

■ PV-02

$$I_{CA} = I_0 \tanh\left(\frac{U_{\text{Diode}}}{2U_T}\right), \quad \cancel{\frac{\Delta I_c}{\Delta U} = I_0\cdot\frac{1}{2U_T}}, \quad U_{10} = \frac{\Delta I_c}{g_m}, \quad g_m = \frac{I_{CA}}{U_T} \Rightarrow U_{10} = U_T\cdot\frac{\Delta I_c}{I_{CA}}$$

$$\sigma(U_{10}) = U_T\,\sigma\!\left(\frac{\Delta I_c}{I_{CA}}\right) \Rightarrow 0.2 = 3U_T\cdot\frac{A_{Ic}}{\sqrt{A_E}} \qquad A_E = \left(\frac{3U_T\cdot A_{Ic}}{0.2}\right)^2 = 136.8\ \mu m^2$$

■ PV-03

$A_{Vth} = 8\,mV\mu m$, $A_\beta = 0.03\,\mu m$, $\sqrt{A} = \sqrt{W\cdot L} = 6\,\mu m$.

$$\sigma(\Delta U_{th}) = \frac{A_{Vth}}{\sqrt{A}} = 1.33V. \quad \sigma\left(\frac{\Delta\beta}{\beta}\right) = \frac{A_\beta}{\sqrt{A}} = 0.005.$$

Aus Vorlesung.

$$\sigma^2\left(\frac{\Delta I_{DA}}{I_{DA}}\right) = \sigma^2\left(\frac{\Delta\beta}{\beta}\right) + \frac{4}{(U_{GS}-U_{th})^2}\sigma^2(\Delta U_{th}) \Rightarrow 0.00832.$$

$$\sigma(U_{IO}) = \frac{1}{g_m}\sigma(\Delta I_{DA}) = \frac{1}{\sqrt{2\beta I_{DA}}}\cdot\sigma\left(\frac{\Delta I_{DA}}{I_{DA}}\right)\cdot I_{DA}. \qquad I_{DA} = \frac{\beta}{2}\cdot(U_{GSA}-U_{th})^2$$

$$= \sqrt{\frac{I_{DA}}{2\beta}}\cdot\sigma\left(\frac{\Delta I_{DA}}{I_{DA}}\right) \qquad dI_{DA} = \beta\cdot dU_{GSA}\cdot(U_{DSA}-U_{th})$$

$$= \sqrt{\frac{(U_{GSA}-U_{th})^2}{4}}\cdot\sigma\left(\frac{\Delta I_{DA}}{I_{DA}}\right) \qquad = \beta\cdot\sqrt{\frac{2I_{DA}}{\beta}} = \sqrt{2I_{DA}\cdot\beta}$$

$$= \frac{0.4}{2}\cdot 0.00832 = 1.67\ mV.$$

■ PV-04

$R_1 = k_1 R_E$. $R_2 = k_2 R_E + \cancel{R_{test}}$ k_1 和 k_2 为整数。 $R_{test}/R_2 < 0.2$. $0 < R_{test} < 15k\Omega$.

$$R_2 = k_2 R_E + R_{rest} = \frac{k_2}{k_1}R_1 + R_{rest},$$

$$\frac{60}{12} < \frac{k_2}{k_1} < \frac{75}{12}, \quad 5 \le \frac{k_2}{k_1} \le 6.25. \quad \Rightarrow$$

假设 $R_{rest} = 0$. $k_2 = 6.25\cdot k_1$, $\Rightarrow k_1 = 4$ 成立.

$\Rightarrow k_1 = 4$, $R_1 = 4R_E$

$k_2 = 25$, $R_2 = 25R_E$, $R_E = 3k\Omega$

■ PV-05

1). $\Delta U_{th} = -k_1\exp(-k_2\cdot n)$ $\Rightarrow$ $n=1$, $\Delta U_{th} = -44.6mV$; $n=3$, $\Delta U_{th} = -2.22mV$.

2). 代入 U_{th}, ans, $n = -\frac{\ln}{k_2}(U_{th,0} - U_{th}/k_1) = 0.824 \Rightarrow L = 231\,\mu m$.

3) $\sigma^2(\Delta U_{th}) = \frac{d\Delta U_{th}}{dL} \Rightarrow \sigma^2(\Delta U) = \frac{k_1 k_2}{L_{min}}\exp(-k_2 n)\sigma(\Delta L)$, $\Rightarrow$ $n=1$, $\delta L = 0.54mV$, $\delta a = 5.36mV$; $n=3$, $\delta L = 0.027mV$, $\delta a = 0.27mV$.

4). 先求 n, $n = 1.657$. $L_s = 415\,nm$.

PV-06. Aus Vorlesung

1) $\sigma^2\left(\frac{\Delta R}{R}\right) = a_1^2\,\sigma^2\left(\frac{\Delta R_1}{R_1}\right) + \dots + a_3^2\,\sigma^2\left(\frac{\Delta R_3}{R_3}\right)$, $a_1+a_2+a_3 = 0$. $\Rightarrow \sigma\left(\frac{\Delta R}{R}\right) = 2.34\% \Rightarrow 3\sigma\left(\frac{\Delta R}{R}\right) = 7.02\%$

2). $R_1 = R\cdot a_1 = 2.18k\Omega$

$R_2 = R\cdot a_2 = 4.81k\Omega$

$R_3 = R\cdot(1-a_1-a_2) = 2.11k\Omega$